A PARTITION CONSTRAINED MINIMIZATION SCHEME FOR EFFICIENT MULTIPHASE LEVEL SET IMAGE SEGMENTATION

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ABSTRACT

This study investigates a new multiphase minimization scheme which embeds a simple, efficient partition constraint directly in multiple level set evolution. Starting from an arbitrary initial partition, the minimization of the *N*-region segmentation functional is carried out following a first order expansion of the data term with an embedded partition constraint: if a point leaves a region, it goes to single other region. The method has a computational advantage over previous multiphase schemes, is stepwise optimal, i.e., permit to effect the maximum decrease of the functional at each evolution step, and is robust to initialization. The method is discussed by comparison with previous methods and experimental results are included to this effect.

1. INTRODUCTION

The variational formalism based on active contours and level sets has recently led to effective segmentation algorithms of difficult images and numerous useful applications [1] [2] [3] [4] [5] [6] [7]. The use of more than one curve required for the segmentation of N regions, with $N > 2$, may lead to ambiguous segmentation results when the interior of curves overlap. The partition problem in multiphase level set segmentation has been addressed in several ways. Zhao et *al.* [9] proposed to add to the functional a term that draws the solution toward a partition. The same principle is also used in [7]. This method does not guarantee a partition at all times. Curve evolution will likely gives an ambiguous segmentation if the partition constraint is not sufficiently enforced, and if it is strongly enforced the curves will evolve more as a result of the partition constraint than of image statistics. This method requires an additional *ad hoc* parameter and there is no clear indication on how to fix this parameter sufficiently without weakening the role of the data term. Authors in [5] proposed a correspondence between the characteristic functions of regions and the different combinations of level set signs such as to guarantee a partition all the time. The method proposed recently in [1], and which we used in [2], establishes an explicit correspondence between the curves, their intersections and the regions of segmentation. Both curve/region correspondence methods [5] [1] embed cumbersome combinations of the characteristic functions of regions in the functional and the resulting PDEs, and consequently the complexity increases significantly with the number of regions as will be discussed in section 2.3. Apart from the high computational complexity when dealing with large number of region, the method in [5] lacks robustness to initialization [5], and with a large number of regions, it becomes difficult to handle initial conditions correctly. Authors in [6] used a functional which results in curve evolution equations where the evolution of a curve involves a reference to the others. However, a segmentation into N regions can be obtained only for vector images of dimension $N-1$ or higher. Also, the observation term in the functional measures an $N-1$ dimensional volume, resulting in excessive computational demand. Authors in [4] proposed to minimizes $N-1$ functionals instead of the initial N-region segmentation functional. The resulting $N-1$ PDEs are demonstrated to guarantee a partition, but it is not clear that they minimizes the initial functional.

In this study, we propose a minimization scheme which embeds directly an efficient simple partition constraint in curve evolution without resorting to cumbersome modifications of the segmentation functional. The method has a computational advantage over previous multiphase schemes, is stepwise optimal, i.e., permits to effect the maximum decrease of the functional at each evolution step, and is robust to initializations. We will discuss the advantages the proposed minimization scheme by comparisons with previous multiphase methods further in section 2.3. Experimental results are included to this effect.

2. FUNCTIONAL MINIMIZATION

Let $I : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^n$ be an image function defined on $\Omega \subset \mathbb{R}^2$. An N-region segmentation of I is a partition $\mathcal{P} = \{\mathbf{R}_i\}_{i\in[1,N]}$ of the image domain such that the restriction of the image to each region best fits a given description, usually given through statistical models. Level set segmentation is commonly stated as the minimization of a functional containing two characteristic terms: a term of conformity to data and a regularization term.

Data term: The data term measures how well the data fits a statistical model within each segmentation region:

$$\mathcal{D} = \sum_{i=1}^{N} \xi_i, \tag{1}$$

where

$$\xi_i = \int_{\mathbf{R}_i} e_i(\mathbf{x})d\mathbf{x}, \tag{2}$$

and e_i is the function which evaluates the conformity to data in region $\mathbf{R}_i$.

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Regularization term: The regularization term permit to obtain smooth segmentation boundaries and to avoid small, isolated segmentation fragments.

$$\mathcal{R} = \sum_{i=1}^{N} \oint_{\partial \mathbf{R}_i} ds. \quad (3)$$

The functional to minimize is a weighed sum of the data and regularization terms:

$$\mathcal{F}(\{\mathbf{R}_i\}_{i=1}^{N}) = \sum_{i=1}^{N} \xi_i + \lambda \sum_{i=1}^{N} \oint_{\partial \mathbf{R}_i} ds. \quad (4)$$

where $\partial \mathbf{R}_i$ is the boundary of $\mathbf{R}_i$ and λ is a positive real constant to weigh the relative contribution of the two terms of the functional.

For a clearer exposition of the algorithm, we treat the binary segmentation problem first (Section 2.1). We generalize to multiple regions in Section 2.3. In the case of N-region segmentation ($N > 2$), we will see that the issue is to embed an efficient partition constraint directly in the functional minimization.

2.1. Curve evolution for binary segmentation

In the case of two regions, we consider a closed planar parametric curve $\vec{\gamma} : [0,1] \to \Omega$. Let $\mathbf{R}_1 = \mathbf{R}_{\vec{\gamma}}$ be the region in the interior of $\vec{\gamma}$, and $\mathbf{R}_2 = \mathbf{R}_1^c$ the region in the exterior.

The minimization of $\mathcal{F}$ with respect to $\vec{\gamma}$ is obtained by embedding the curve $\vec{\gamma}(s)$ in a one-parameter family of curves: $\vec{\gamma}(s,t) : [0,1] \times \mathbf{R}_+ \to \Omega$, and solving the partial differential equation (PDE):

$$\frac{d\vec{\gamma}}{dt} = -\frac{\partial \mathcal{F}}{\partial \vec{\gamma}} = -\frac{\partial \mathcal{D}}{\partial \vec{\gamma}} - \frac{\partial \mathcal{R}}{\partial \vec{\gamma}}, \quad (5)$$

where $\frac{\partial \mathcal{F}}{\partial \vec{\gamma}}$ is the functional derivative of $\mathcal{F}$ with respect to $\vec{\gamma}$.

To derive the data term with respect to $\vec{\gamma}$, we propose a computation based on its first order expansion rather than the standard Euler-Lagrange equation. This leads to the same velocity expression as the Euler-Lagrange equation but has the advantage of leading to a clear interpretation of how to embed a simple, efficient partition constraint directly in the multiphase minimization scheme.

Let $\vec{\delta\gamma} = (\delta x, \delta y)^T = \delta\gamma.\vec{n}$ be an elementary local deformation of $\vec{\gamma}$ ($||\vec{\delta\gamma}|| = 1$) around a pixel $s = (x,y)$, where $\vec{n}$ is the external unit normal to $\vec{\gamma}$ at $s = (x,y)$. The first order expansion of $\mathcal{D}$ gives:

$$\begin{aligned} \mathcal{D}(\vec{\gamma} + \vec{\delta\gamma}) &= \mathcal{D}(x + \delta x, y + \delta y) \\ &= \mathcal{D}(x,y) + \frac{\partial \mathcal{D}}{\partial x}\delta x + \frac{\partial \mathcal{D}}{\partial y}\delta y \\ &= \mathcal{D}(\vec{\gamma}) + \frac{\partial \mathcal{D}}{\partial \vec{\gamma}}.\vec{\delta\gamma}. \end{aligned} \quad (6)$$

Multiplying each side in (6) by $\vec{\delta\gamma}$ yields:

$$\frac{\partial \mathcal{D}}{\partial \vec{\gamma}}.||\vec{\delta\gamma}|| = \frac{\partial \mathcal{D}}{\partial \vec{\gamma}} = \left(\mathcal{D}(\vec{\gamma} + \vec{\delta\gamma}) - \mathcal{D}(\vec{\gamma})\right).\vec{\delta\gamma} \quad (7)$$

Let $\delta a_{\mathbf{R}}$ is the elementary variation of the area of region $\mathbf{R}$. $\delta a_{\mathbf{R}} = 1$ if the curve is locally expanding around a pixel $s = (x,y)$ to contain it, and $\delta a_{\mathbf{R}} = -1$ when the curve is shrinking. Let $\Delta\xi_i(s)$ be the elementary variation of ξ_i. Consider the local variation $\Delta\mathcal{D}(s) = \mathcal{D}(\vec{\gamma} + \vec{\delta\gamma}) - \mathcal{D}(\vec{\gamma})$. We have:

$$\begin{aligned} \Delta\mathcal{D}(s) &= \Delta\xi_1(s) + \Delta\xi_2(s) \\ &= \delta a_{\mathbf{R}_1} e_1(s) + \delta a_{\mathbf{R}_2} e_2(s) \end{aligned} \quad (8)$$

Suppose, without loss of generality, that $\vec{\gamma}$ is expanding to contain s, i.e., $\delta a_{\mathbf{R}_1} = 1$ and $\delta a_{\mathbf{R}_2} = -\delta a_{\mathbf{R}_1} = -1$ because $\mathbf{R}_2 = \mathbf{R}_1^c$, then:

$$\begin{aligned} \Delta\xi_1(s) &= \Delta\xi_1^+(s) = e_1(s) \\ \Delta\xi_2(s) &= \Delta\xi_1^-(s) = -e_2(s) \end{aligned} \quad (9)$$

$\Delta\xi_i^+(s)$ is the variation of the data term $\mathcal{D}$ corresponding to the variation of ξ_i when a pixel s enters the region $\mathbf{R}_i$. $\Delta\xi_i^-(s)$ is the variation of the data term $\mathcal{D}$ corresponding to the variation of ξ_i when a pixel s leaves the region $\mathbf{R}_i$.

Using (9), (8), and (7) gives:

$$\begin{aligned} \frac{\partial \mathcal{D}}{\partial \vec{\gamma}} &= \left(\Delta\xi_1^+(s) - \Delta_{\xi_2}^+(s)\right).\vec{\delta\gamma} \\ &= \left(e_1(s) - e_2(s)\right).\vec{\delta\gamma} \end{aligned} \quad (10)$$

Suppose, without loss of generality, that $\vec{\gamma}$ is expanding to contain s, i.e., $\delta a_{\mathbf{R}_1} = 1$ and $\delta a_{\mathbf{R}_2} = -\delta a_{\mathbf{R}_1} = -1$ because $\mathbf{R}_2 = \mathbf{R}_1^c$. Then, we have: $\vec{\delta\gamma} = \vec{n}$, where $\vec{n}$ is the external unit normal to $\vec{\delta\gamma}$ at point s. Therefore, the functional derivative of $\mathcal{D}$ with respect to $\vec{\gamma}$ is:

$$\frac{\partial \mathcal{D}}{\partial \vec{\gamma}} = \left(\Delta\xi_1^+(s) - \Delta_{\xi_2}^+(s)\right).\vec{n} \quad (11)$$

When $\vec{\gamma}$ is shrinking at s, we have $\Delta\xi_1(s) = -\Delta\xi_1^+(s) = -e_1(s)$, $\Delta\xi_2(s) = \Delta\xi_2^+(s) = e_2(s)$, and $\vec{\delta\gamma} = -\vec{n}$. Thus, when $\vec{\gamma}$ is shrinking at s we obtain the same expression of the velocity as in (11).

The computation of $\frac{\partial \mathcal{R}}{\partial \vec{\gamma}}$ is classical and follows the standard calculus of Euler-Lagrange equations [10]:

$$\frac{\partial \mathcal{R}}{\partial \vec{\gamma}} = \frac{\partial \lambda \oint_{\vec{\gamma}} ds}{\partial \vec{\gamma}} = -\lambda\kappa\vec{n} \quad (12)$$

where κ is the mean curvature function of $\vec{\gamma}$. The final evolution equation of $\vec{\gamma}$ is given by:

$$\frac{\partial \vec{\gamma}}{\partial t} = -\left(\Delta\xi_1^+(s) - \Delta_{\xi_2}^+(s) + \lambda\kappa\right).\vec{n} \quad (13)$$

2.2. level set implementation

We use the level set representation [8] to implement the evolution equation (13). With the level set representation, curve $\vec{\gamma}$ is represented implicitly as the zero level-set of a function $u : \mathbb{R}^2 \to \mathbb{R}$, i.e., $\vec{\gamma}$ is the set $\{u = 0\}$. The level set evolution equation corresponding to (13) is [8]:

$$\frac{\partial u}{\partial t}(\mathbf{x}, t) = -\left(\Delta\xi_1^+(\mathbf{x}) - \Delta\xi_2^+(\mathbf{x}) + \lambda\kappa_u\right)\|\vec{\nabla} u(\mathbf{x}, t)\| \quad (14)$$

where $\Delta\xi_i^+(\mathbf{x}) = e_i(\mathbf{x}), i = 1, 2, \forall \mathbf{x} \in \mathbf{R}_i$. κ_u is the curvature of $\{u = 0\}$.

2.3. Extension to multiphase segmentation

Consider a family of simple closed curves $\vec{\gamma}_k|_{k=1,\dots,N-1}$ and let $\mathbf{R}_k = \mathbf{R}_{\vec{\gamma}_k}|_{k=1,\dots,N-1}$. Let $\mathbf{R}_N = \bigcap_{i=1}^{N-1} \mathbf{R}_i^c$. We will impose a simple, efficient partition constraint directly on the multiphase curve evolution as follows:

Partition constraints: 1) Start from an initial partition $\mathcal{P}^0 = \{\mathbf{R}_k^0\}_{k\in[1,N]}$.

2) Suppose we have a partition $\mathcal{P}^t = \{\mathbf{R}_k^t\}_{k\in[1,N]}$ at iteration t, and let $\mathbf{x} \in \Omega$. If $\mathbf{x} \in \mathbf{R}_i^t, i \in [1,..,N]$ and $\mathbf{x}$ leaves region $\mathbf{R}_i^t$, it must move to another region $\mathbf{R}_j, j \in [1..N], j \neq i$, and only one other region, i.e., $\mathbf{x} \in \mathbf{R}_j^{t+1}$ and $\forall k \neq j, \mathbf{x} \notin \mathbf{R}_k^{t+1}$. To satisfy condition 2), the curve evolution equations at pixel $\mathbf{x}$ must involve at most two curves, i.e., only two regions: region $\mathbf{R}_i$ which contains pixel $\mathbf{x}$ and another region $\mathbf{R}_j, j \neq i$. To obtain the multiphase curve evolution equations satisfying 2), we fix curves $\vec{\gamma}_k, k \notin \{i, j\}$, and minimize the functional with respect to the variation of $\vec{\gamma}_i$ if $i \neq N$, and $\vec{\gamma}_j$ if $j \neq N$, i.e.,

$$
\begin{aligned}
&\text{if } i \neq N, \frac{\partial \mathcal{F}}{\partial \vec{\gamma}_i} = \frac{\partial\left(\xi_i + \xi_j\right)}{\partial \vec{\gamma}_i} + \frac{\partial \lambda \oint_{\vec{\gamma}_i} ds}{\partial \vec{\gamma}_i} \\
&\text{if } j \neq N, \frac{\partial \mathcal{F}}{\partial \vec{\gamma}_j} = \frac{\partial\left(\xi_i + \xi_j\right)}{\partial \vec{\gamma}_j} + \frac{\partial \lambda \oint_{\vec{\gamma}_j} ds}{\partial \vec{\gamma}_j}
\end{aligned}
\tag{15}
$$

Therefore, multiphase segmentation reduces to a binary problem corresponding to the variation $\Delta_{\mathbf{R}_i} + \Delta_{\mathbf{R}_j}$ of $\mathcal{D}$ in the domain $\mathbf{R}_i \cup \mathbf{R}_j$. Following the computation in the two-region case, the level-set curve evolution equations corresponding to the minimization of $\mathcal{F}$ with respect to $\vec{\gamma}_i$, if $i \neq N$, and with respect to $\vec{\gamma}_j$, if $j \neq N$ are given by:

$$
\begin{aligned}
&\text{if } i \neq N, \\
&\frac{\partial u_i}{\partial t}(\mathbf{x}, t) = -\left(\Delta^+_{\xi_i}(\mathbf{x}) - \Delta^+_{\xi_j}(\mathbf{x}) + \lambda \kappa_{u_i}\right) \|\vec{\nabla} u_i(\mathbf{x}, t)\| \\
&\text{if } j \neq N, \\
&\frac{\partial u_j}{\partial t}(\mathbf{x}, t) = -\left(\Delta^+_{\xi_j}(\mathbf{x}) - \Delta^+_{\xi_i}(\mathbf{x}) + \lambda \kappa_{u_j}\right) \|\vec{\nabla} u_j(\mathbf{x}, t)\|
\end{aligned}
\tag{16}
$$

where u_k is the level-set function corresponding to $\vec{\gamma}_k, k \in [1..N-1]$, and κ_{u_k}, is the curvature of the zero level-set of u_k. It is clear that the curve evolution equations defined in system (16) satisfy the partition condition 2). If $i = N$ or $j = N$, the system (16) is equivalent to only one evolution equation corresponding to the two-region segmentation problem in the domain $\mathbf{R}_i \cup \mathbf{R}_j$. If $i \neq N$ and $j \neq N$ and if we ignore the contribution of the curvature term, the two evolving curves $\vec{\gamma}_i$ and $\vec{\gamma}_j$ have opposite velocities at point $\mathbf{x}$. Thus, if $\vec{\gamma}_i$ shrinks at $\mathbf{x}$, $\vec{\gamma}_j$ expands to contain it and vice versa. If the contribution of the curvature term is important, both evolving curves shrink and $\mathbf{x}$ leaves the interior of one curve to enter the background region $\mathbf{R}_N$.

definition of $\mathbf{R}_j$ The problem now is the definition of the region $\mathbf{R}_j, j \in [1..N], j \neq i$, that will be involved in system (16) at a given pixel $\mathbf{x} \in \Omega$.

Let $\mathbf{x} \in \mathbf{R}_i$ and suppose $\mathbf{x}$ leave $\mathbf{R}_i$ to enter $\mathbf{R}_j, j \in [1..N], j \neq i$. The resulting variation of the data term $\mathcal{D}$ is $\Delta^+_{\mathbf{R}_j}(\mathbf{x}) - \Delta^+_{\mathbf{R}_i}(\mathbf{x})$. Since we aim to minimize $\mathcal{F}$, the best variation is given by:

$$
\begin{aligned}
j_0 &= \arg \min_{\{j\in[1..N],\ \mathbf{x}\notin\mathbf{R}_j\}} (\Delta^+_{\mathbf{R}_j}(\mathbf{x}) - \Delta^+_{\mathbf{R}_i}(\mathbf{x})) \\
&= \arg \min_{\{j\in[1..N],\ \mathbf{x}\notin\mathbf{R}_j\}} \Delta^+_{\mathbf{R}_j}(\mathbf{x})
\end{aligned}
\tag{17}
$$

This leads to the following multiphase level set equations, for all $\mathbf{x} \in \Omega$: $\forall i \in [1..N], if\ \mathbf{x} \in \mathbf{R}_i,$ do

$$
\begin{aligned}
&\text{if } i \neq N, \\
&\frac{\partial u_i}{\partial t}(\mathbf{x}, t) = -\left(\Delta^+_{\xi_i}(\mathbf{x}) - \Delta^+_{\xi_{j_0}}(\mathbf{x}) + \lambda \kappa_{u_i}\right) \|\vec{\nabla} u_i(\mathbf{x}, t)\| \\
&\text{if } j_0 \neq N, \\
&\frac{\partial u_{j_0}}{\partial t}(\mathbf{x}, t) = -\left(\Delta^+_{\xi_{j_0}}(\mathbf{x}) - \Delta^+_{\xi_i}(\mathbf{x}) + \lambda \kappa_{u_{j_0}}\right) \|\vec{\nabla} u_{j_0}(\mathbf{x}, t)\|
\end{aligned}
\tag{18}
$$

where $i \in [1..N]$ is the index of the region containing $\mathbf{x}$ and j_0 is given by (17).

As with previous multiphase methods [5][1][9] [7], this method converges to a local minimum since it is based on gradient descent. However, it is stepwise optimal because it effects the maximum decrease in the functional at each curve evolution step. This comes directly from the definition of j_0 in equation (17). We will give in section 3 experimental illustrations of the stepwise optimality.

This multiphase method has a computational advantage over the methods in [5][1][9] [7]. It activates at most two PDEs at each iteration. The CPU time varies approximately linearly with the number of regions due to the search for index j_0. The methods in [5][1] [9][7] activate the PDEs corresponding to all the level sets at each iteration and the complexity of the corresponding PDEs increases with the number of regions . The methods in [5][1] also evaluate an expensive point membership function. For the method in [1], this involves, for a given level set, checking the sign of all lower numbered level sets. For the method in [5], this involves checking the signs of all level set intersections. This results in a variation of the computation time versus the number of regions faster than linear. Figure 3 a) illustrates this. It shows the CPU time spent at an iteration as a function of the number of regions for this method and the method in [1]. The growth of the curve for the method in [5] would be similar to the one for the method in [1] or steeper.

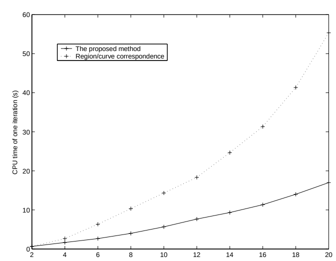

Fig. 1. a) CPU time versus the number of regions.

3. EXPERIMENTATION

The proposed minimization scheme has been tested in several experiments and the results are conclusive. In the following, we present some representative results using gray level images and the piecewise constant model. The first example uses the brain image shown in Figure 2 a) with initial curves. We show the final position of the curves in Figure 2 b), the final segmentation in Figure 2 c), and three of the four regions of segmentation in Figure

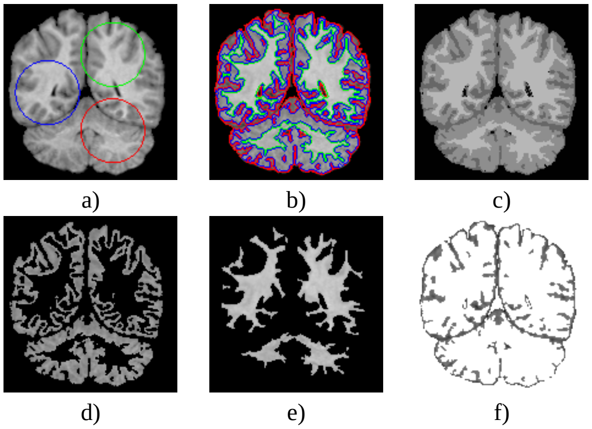

Fig. 2. Results with the brain image: a) initial curves, b) final curves, c) segmentation, d), e), and f) regions of segmentation

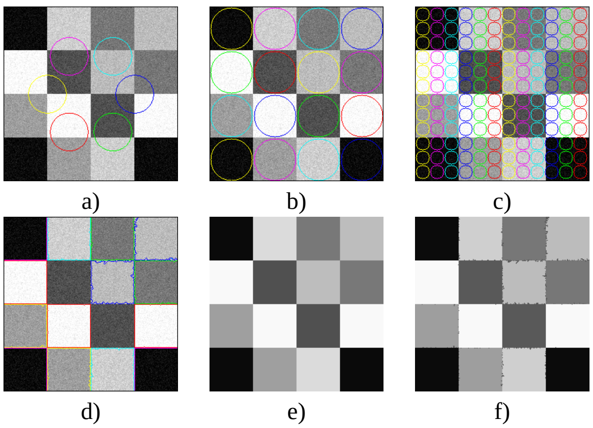

Fig. 3. Results with the synthetic image: a), b), and c) different initializations, d) final curves, e) true segmentation, and f) obtained segmentation

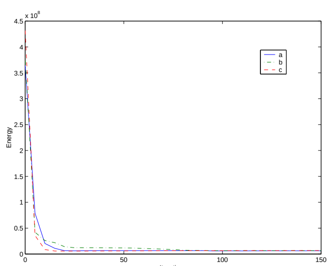

Fig. 4. The minimized energies versus the number of iterations

2 d), e) and f). The obtained partition is conform to our expectation. Due to the stepwise optimality, the method requires a number of iterations less than previous methods. With the example of the brain image, and using the same parameters and initialization, the proposed minimization scheme requires about 60 iterations to converge, whereas the method in [1] requires about 400 iterations.

To illustrate the robustness of the method with respect to initial conditions and its ability to deal efficiently with a large number of regions, we use the 7-region image in Figure 3 (with initial curves). The contrast between some regions is low and a Gaussian noise was added to this image. We tested the three different initializations shown in Figure 3 a), b) , and c), and plotted the minimized energies versus the number of iterations in Figure 4. For both initializations, the evolution of the minimized energies has almost the same behavior and the same minimum is reached after about 50 iterations.This is also due to the stepwise optimality and demonstrates the robustness of the method to initial conditions. We show the final position of curves in Figure 3 d), the obtained segmentation in 3 f), and the true segmentation in 3 f).

4. CONCLUSION

We described a minimization scheme which embeds an efficient simple partition constraint directly in multiple level set evolution, and without resorting to modifying the segmentation functional by a partition term or cumbersome region/curve correspondences. We included experimental results which demonstrated the advantages of the proposed minimization scheme over previous methods.

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